GCE

Further Mathematics B MEI

Y412/01: Statistics A

AS Level

Mark Scheme for June 2024

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

MARKING INSTRUCTIONS

PREPARATION FOR MARKING
RM ASSESSOR

1.   Make sure that you have accessed and completed the relevant training packages for on-screen marking: *RM Assessor Online Training*; *OCR Essential Guide to Marking*.

2.   Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3.   Log-in to RM Assessor and mark the **required number** of practice responses ("scripts") and the **number of required** standardisation responses.

MARKING

1.   Mark strictly to the mark scheme.

2.   Marks awarded must relate directly to the marking criteria.

3.   The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4.   If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone or the RM Assessor messaging system, or by email.

5. **Annotations**

Annotation	Meaning
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.
BP	Blank Page
Seen	
Highlighting	

6. **Subject Specific Marking Instructions**

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

   For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

   It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

   Award No Response (NR) if:
   - there is nothing written in the answer space

   Award Zero '0' if:
   - anything is written in the answer space and is not worthy of credit (this includes text and symbols).

   Team Leaders must confirm the correct use of the NR button with their markers before live marking commences and should check this when reviewing scripts.

   If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

   If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

PMT

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)
    We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
    • When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
    • When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
      NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f".

    Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
    Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
    • If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
    • If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
    • if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
    If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i.   If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j.   If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		$2c + 3c + 0.5 - c + c = 1$ $\Rightarrow\ 0.5 + 5c = 1$ $\Rightarrow c = 0.5/5 = 0.1$	M1 A1 **[2]**	1.1 1.1	Using $\Sigma p = 1$.	
1	(b)		$E(X) = 1.4$ cao $Var(X) = (0^2 \times 0.2\ +)\ 1^2 \times 0.3 + 2^2 \times 0.4 + 3^2 \times 0.1$ $\qquad - \text{``}1.4\text{''}^2$ or $(0^2 \times 2c\ +)\ 1^2 \times 3c + 2^2(0.5 - c) + 3^2 c - \text{``}1.4\text{''}^2$ $Var(X) = 2.8 - 1.4^2 = 0.84$	B1 M1 A1FT **[3]**	1.1 1.1 1.1	 FT their c provided $Var(X) > 0$	Could see $0 \times 2c + 1 \times 3c + 2(0.5 - c) + 3c$ $(= 1 + 4c)$ or $0 \times 0.2 + 1 \times 0.3 + 2 \times 0.4 + 3 \times 0.1$ Correct form for finding variance, either numerical or in terms of c. NB $Var(X) = 2 + 8c - (1 + 4c)^2$ If **B0** because $E(X) = 1 + 4c$ then **A1** can be awarded for $Var(X) = (1 + 4c)(1 - 4c)$ or $1 - 16c^2$
1	(c)		$E(Y) = -0.2$ $Var(Y) = 3.36$	B1FT B1FT **[2]**	1.1 1.1	FT $2 \times \text{``}1.4\text{''} - 3$ FT $2^2 \times \text{``}0.84\text{''}$ provided $Var(X) > 0$	**SC1** for both $E(Y) = 8c - 1$ and $Var(Y) = 4 - 64c^2$ oe

Question			Answer	Marks	AO	Guidance	
2	**(a)**		(Discrete) Uniform or U... on the values 1, 2, …, 32 **[2]**	**M1** **A1**	**1.1a** **2.5**	Correct distribution stated. Condone "1 to 32", or equivalent, provided it is clear that this is a discrete distribution.	
2	**(b)**		1/32 or 0.03125 or 3.125×10^{-2} **[1]**	**B1**	**1.1**	Exact value seen.	
2	**(c)**		$Y \sim \text{Geo}(1/32)$ $P(Y \le 15) = 1 - (31/32)^{15}$ = awrt 0.379 **[3]**	**M1** **M1** **A1**	**1.1a** **1.1** **1.1**	Correct geometric distribution used. soi. Any correct formulation of the required probability seen. If **M1M1** not awarded, **SCB1** for awrt 0.379	eg $\left(\frac{31}{32}\right)^{14} \times \frac{1}{32} + \left(\frac{31}{32}\right)^{13} \times \frac{1}{32} + \ldots + \frac{1}{32}$
2	**(d)**		E(Y) = 32 soi So expected final score = $-1 \times 31 + 15$ = -16 **[3]**	**B1** **M1** **A1**	**1.1** **3.1b** **1.1**	Using E(Y) = 1/p for Geometric Distribution. For $-1 \times$ ("32" $- 1$) + 15. Method seen. If **M0** then **SCB1** for -16	or expected score per turn = $-1 \times (31/32) + 15 \times (1/32)$ (= $-1/2$) E($-Y/2$) = $-32/2 = -16$ oe for **B1A1**

Question			Answer	Marks	AO	Guidance	
3	(a)		mean = 84/40 = 2.1 variance: $\sigma_{n-1}^2 = (256 - 40 \times 2.1^2)/39$ = awrt 2.04 The mean and variance have a similar value. (In a Poisson distribution the mean and variance are equal so this supports the Poisson model suggestion.) [3]	B1 B1FT B1	1.1 1.1 3.3	 FT their mean Provided their mean and variance both round to 2.	 Allow $2.1 \approx 2.04$ or mean $\approx$ variance.
3	(b)	(i)	<table><tr><td>Prob</td><td>EF</td><td>χ^2 cont</td></tr><tr><td>0.13534</td><td>5.4134</td><td>2.15232</td></tr><tr><td>**0.27067**</td><td>**10.8268**</td><td>**0.93001**</td></tr><tr><td>0.27067</td><td>**10.8268**</td><td>0.43620</td></tr><tr><td>**0.18045**</td><td>7.2179</td><td>**0.68150**</td></tr><tr><td>0.14288</td><td>**5.7151**</td><td>0.01421</td></tr></table> [4]	M1 A1 M1 A1	1.1 1.1 1.1 1.1	At least 1 EF correct to at least 2d.p. EF column all correct to at least 2d.p. At least 1 contribution correct to at least 3d.p. Contributions both correct to at least 3d.p.	If **M0M0** then **SC1** for at least one probability correct to 3 s.f. shown in the table.
3	(b)	(ii)	H_0: The Poisson model fits the data H_1: The Poisson model does not fit the data χ^2 = awrt 4.21 $v = 4$ $(\chi^2_4)_{10\%}$= 7.779 4.21 < 7.779 so the result is not significant we do not reject H_0 oe There is insufficient evidence at the 10% level to suggest that the Poisson model does not fit the data. [6]	B1 B1FT M1 A1 M1 A1	3.3 3.4 1.1 1.1 2.2b 3.5a	or H_0: The Poisson model is suitable H_1: The Poisson model is not suitable FT for 2.60273 + their contributions Can be implied by 7.779 For correctly comparing their test statistic with their critical value and then making a consistent conclusion. From correct values and hypotheses only. Contextual and non-assertive. An assertive "post-conclusion" is **A0**.	Condone "the Poisson model is a good fit" or "the given model …" Can be implied by p-value = 0.378 **A1** for p-value = 0.378 or p-value = awrt 0.378 > 0.1 Allow "It is reasonable to believe that the Poisson model is suitable"

3	(c)		These defects did not occur independently of each other [1]	B1	3.5b	or these defects did not occur singly.	
Question			**Answer**	**Marks**	**AO**	**Guidance**	
4	(a)		t should go on the horizontal axis as it is the independent/control/non-random variable. [1]	B1	1.2		
4	(b)	(i)	$S_{tt} = 41400 - \dfrac{360^2}{5}\,(=15480)$ or $S_{tz} = 5835 - \dfrac{360 \times 123}{5}\,(=-3021)$	M1	1.1	$41400 - 5 \times 72^2$ or $5835 - 5 \times 72 \times 24.6$ For calculation for S_{tt} or S_{tz} seen.	For reference, $S_{zz} = 3629.74 - \dfrac{123^2}{5} = 603.94$
			$(b=)\dfrac{S_{tz}}{S_{tt}} = \dfrac{-3021}{15480} = -0.1951550...\ = $ awrt -0.195	A1	1.1	$-1007/5160$	awrt -0.195 with no working **SCB1**
			$(a =)\ 24.6 - \text{“}-0.195\text{”} \times 72$ oe	M1	1.1	For using $a = \bar{z} - b\bar{\bar{t}}$ oe	
			$a =$ awrt 38.7 [4]	A1	1.1	1662/43 Condone $a = 38.6$	$a =$ awrt 38.7 with no working **SCB1**
4	(b)	(ii)	$t = 60$: awrt 0.56 $t = 120$: awrt -2.4 [2]	B1FT B1FT	1.1 1.1	FT their a and b. Answer to $\geq$ 2s.f. FT their a and b. Answer to $\geq$ 2s.f.	**SCB1** Both values correct in magnitude but both signs incorrect.
4	(c)	(i)	$z(90) \approx 38.65... - 0.1951... \times 90 = 21.08...$ So estimate of concentration is 21 mg cm^{-3} [1]	B1FT	3.4	FT their a and b. Answer to 1s.f. 2s.f. or 3s.f. only. Answers of 4s.f. or more are over-specified.	.
4	(c)	(ii)	(This is interpolation and) the residuals (for $t =$ 60 and 120) are (reasonably) small (in relation to the values) so the estimate is likely to be reliable. [1]	B1	3.5a	Condone over-assertive answers (eg "so the estimate **is** reliable").	Accept a different sensible conclusion relating to their residuals. eg "Even though this is interpolation and surrounding residuals are small the other residuals may be large so we can't tell."

4	(d)		$t = 200$ gives $z = -0.38$ so for $t > 200$ the model predicts that the concentration is negative, which is impossible. **[1]**	**B1**	**3.5b**	For sensible comment supported with a numerical example.	Stating that it is (further) extrapolation is, in itself, insufficient for **B1**. **B0** if their gradient is positive.

Question			Answer	Marks	AO	Guidance	
5	(a)		The underlying distribution is bivariate normal **[1]**	**B1**	**1.2**	Context not required.	Not "normal bivariate". Not "the data is bivariate normal".
5	(b)		$H_0: \rho = 0$ $H_1: \rho \neq 0$ where ρ is the **population** correlation coefficient between amount of coffee and number of waking hours $(\pm)\ 0.7067$ $(-0.7067 <)\ 0.6030 < 0.7067$ so we do not reject H_0 oe There is insufficient evidence at the 5% level to suggest that there is any **correlation** between volume of coffee drunk (in a day) and number of hours remaining awake (that day) **[5]**	**B1** **B1** **B1** **M1** **A1**	**3.3** **2.4** **3.4** **2.2b** **3.5a**	Allow hypotheses in words provided these refer to the population correlation coefficient. Defining ρ contextually. CV for $n = 8$, 2-tailed test at 5% For correctly comparing their test statistic with their critical value and then making a consistent conclusion For non-assertive conclusion in context that refers to H_1, from correct CV and hypotheses only.	If one-tailed test used allow CV = 0.6215 for **B1FT** or "the result is not significant"
5	(c)		(The new test statistic is 0.5487 and) the new CV is 0.3610, $(0.5487 > 0.3610)$ so the result will be significant oe e.g. H_0 is rejected **[2]**	**M1** **A1**	**2.2a** **2.2a**	Identifying new CV Condone an assertive or non-contextual conclusion here	If one-tailed test used allow CV = 0.3061 for **M1**. **A1FT** available.

PMT

5	(d)		The second test is based on a **larger sample** so it is likely to be more reliable.	**B1**	2.1	This observation alone is sufficient for **B1**.	
			The **effect size is large** so this test is **informative** oe	**B1**	2.4	The idea that the fact that the pmcc is high enough for the test to be significant is not just caused by random factors	Allow "meaningful" for "informative".
				[2]			

Question			Answer	Marks	AO	Guidance
6	(a)	(i)	B or bin or binomial with $n = 10$ and $p = 0.43$ A trial comprises looking at a withdrawal and determining whether it is small or large. There is a fixed number of these and each must result in exactly one of these two outcomes (with constant probability). **[3]**	M1 A1 B1	1.1a 3.3 2.1	B(10, 0.43) is **M1A1** Fixed number of trials, each trial has only two possible outcomes.
6	(a)	(ii)	We must assume that each withdrawal is independent of other withdrawals. or We must assume that the 10 withdrawals are (or sample is) selected randomly. **[1]**	B1	2.4	Must refer to withdrawals.
6	(b)	(i)	awrt 0.246 **[1]**	B1	1.1	0.2462307291...
6	(b)	(ii)	awrt 0.140 **[1]**	B1	1.1	0.1401294608...
6	(b)	(iii)	If X is the no. small withdrawals, $P(X \geq 6)$ $1 - 0.7792942284 =$ awrt 0.221 **[2]**	M1 A1	3.1b 1.1	Or $P(Y \leq 4)$ where Y is the no. large. Can be implied by correct answer. 0.2207057716... (could be direct from B(10, 0.57))
6	(c)		Use of B(9, 0.43) or B(9, 0.57) $0.57 \times 0.22905... =$ awrt 0.131 **[2]**	M1 A1	3.1b 1.1	Can be implied by correct answer.

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